EXERCISE – V

JEE PROBLEMS

1. (a) If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) & (x_3, y_3) **[JEE 99, 2 + 3 + 10]**

(A) lie on a straight line (B) lie on an ellipse

(C) lie on a circle (D) are vertices of a triangle.

Sol.

(b) On the ellipse, $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are

(A) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (B) $\left(-\frac{2}{5}, \frac{1}{5}\right)$ (C) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (D) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

Sol.

(c) Consider the family of circles, $x^2 + y^2 = r^2$, $2<r<5$. If in the first quadrant, the common tangent to a circle of the family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinates axes at A & B, then find the equation of the locus of the mid-point of AB.

Sol.

2. Find the equation of the largest circle with centre (1,0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$.

Sol. **[REE 99, 6]**

3. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from A, B, C to the major axis of the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, (a > b) meet the ellipse respectively at P, Q, R so that P, Q, R lie on the same side of the major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P, Q and R are concurrent.
Sol. **[JEE 2000, 7]**

4. Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C.
Sol. **[JEE 2001, 5]**

5. Find the condition so that the line px + qy = r intersects the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in points whose eccentric angles differ by $\frac{\pi}{4}$. **[REE 2001, 3]**

Sol.

6. Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. **[JEE 2002, 5]**
Sol.

7. (a) The area of the quadrilateral formed by the tangents at the ends of the latus rectum of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is **[JEE 2003 (Scr.)]**

(A) $9\sqrt{3}$ sq. units (B) $27\sqrt{3}$ sq. units

(C) 27 sq. units (D) none

Sol.

(b) The value of θ for which the sum of intercept on the axis by the tangent at the point $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$, $0 < \theta < \pi/2$ on the ellipse $\frac{x^2}{27} + y^2 = 1$ is least, is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{8}$

Sol.

8. The locus of the middle point of the intercept of the tangents drawn from an external point to the ellipse $x^2 + 2y^2 = 2$, between the coordinates axes, is

(A) $\frac{1}{x^2} + \frac{1}{2y^2} = 1$ (B) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

(C) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (D) $\frac{1}{2x^2} + \frac{1}{y^2} = 1$

Sol. **[JEE 2004 (Scr.)]**

9. (a) The minimum area of triangle formed by the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and coordinate axes is **[JEE 2005 (Scr.)]**

(A) ab sq. units (B) $\frac{a^2+b^2}{2}$ sq. units

(C) $\frac{(a+b)^2}{2}$ sq. units (D) $\frac{a^2+ab+b^2}{3}$ sq. units

Sol.

(b) Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Also find the length of the intercept of the tangent between the coordinate axes.

Sol. **[JEE 2005 (Mains), 4]**

10. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are **[JEE 2008, 4]**

(A) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$ (B) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

(C) $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$ (D) $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

Sol.

11. The line passing through the extremity A of the major axis of extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets the auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is **[JEE 2009]**

(A) 31/10 (B) 29/10 (C) 21/10 (D) 27/10

Sol.

12. The normal at a point P on the ellipse $x^2+4y^2=16$ meets the x-axis at Q. If M is the midpoint of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points **[JEE 2009]**

(A) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2},\pm\sqrt{\frac{19}{4}}\right)$

(C) $\left(\pm2\sqrt{3},\pm\frac{1}{7}\right)$ (D) $\left(\pm2\sqrt{3},\pm\frac{4\sqrt{3}}{7}\right)$

Sol.

Paragraph for questions 13 to 15

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B

[JEE 2010]

13. The coordinates of A and B are

(A) (3, 0) and (0, 2) (B) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5},\frac{8}{5}\right)$

(C) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and (0,2) (D) (3, 0) and $\left(-\frac{9}{5},\frac{8}{5}\right)$

Sol.

14. The orthocenter of the triangle PAB is

(A) $\left(5,\frac{8}{7}\right)$ (B) $\left(\frac{7}{5},\frac{25}{8}\right)$ (C) $\left(\frac{11}{5},\frac{8}{5}\right)$ (D) $\left(\frac{8}{25},\frac{7}{5}\right)$

Sol.

15. The equation of the locus of the point whose distances from the point P and the line AB are equal is

(A) $9x^2+y^2-6xy-54x-62y+241=0$

(B) $x^2+9y^2+6xy-54x+62y-241=0$

(C) $9x^2+9y^2-6xy-54x-62y-241=0$

(D) $x^2+y^2-2xy+27x+31y-120=0$

Sol.